ECE 536 – Spring 2022

Homework #5 – Solutions

Problem 1)

First, we relate the optical power to the relevant parameters

$$S = \frac{E^2}{\omega}\frac{k_{op}}{2\mu} = \frac{E^2 n_r}{2\mu c} = \frac{P_{out}}{A} = 10mW/A \tag{5.1}$$

(A) FIELD MAGNITUDE The above equation is easily rearranged to provide an expression for the field magnitude such that $E = \sqrt{2S\mu_o c/n_r} = 0.33\text{MV/m}$. k_{op} is $2\pi n_r/\lambda$, which gives $2.67\times10^7\text{m}^{-1}$. To find the A_0, we use the relation given at the beginning given that $|E| = |i\omega A_0|$, which yields $A_0 = 1.413\times10^{-10}\text{Vs/m}$.

(B) 1.55 μM If we increase the optical wavelength to 1.55 μm, the field magnitude E remains unchanged, but the optical k-number decreases to $1.38\times10^7\text{m}^{-1}$ and A_0 increases to $2.738\times10^{-10}\text{Vs/m}$.

Problem 2)

I. GRAPHICAL METHOD Plot the transcendental along with the radius to find that the first conduction band energy is 30.7meV. Then, we can use

$$L_{eff} = \left[\frac{2m_e^* E_{c1}}{\pi^2\hbar^2}\right]^{-2} \tag{2.1}$$

to see that the effective well width is 13.6nm. A plot showing the intersection is shown in Fig. 2.1(a).

II. ENERGY AND MOMENTUM MATRIX Since we use the infinite barrier model for the other energy levels, $E_{c2} = 4E_{c1}$ =122.8meV. For the momentum matrix element, we have

$$\begin{aligned} M_{21} &= |e|\int_0^{L_{eff}} \phi_1(z) z \phi_2(z)\, dz \\ &= |e|\frac{2}{L_{eff}}\int_0^{L_{eff}} z\sin\left(\frac{\pi}{L_{eff}}z\right)\sin\left(\frac{2\pi}{L_{eff}}z\right) dz \end{aligned} \tag{2.2}$$

This integral can either be found in an integral table or solved by hand using the product-to-sum formula and integration by parts. Either way,

$$M_{21} = \frac{-16}{9\pi^2}|e|L_{eff} \tag{2.3}$$

The expression evaluates to -3.92$\times10^{-18}$CA for this geometry.

III. ABSORPTION SPECTRUM To see the absorption spectrum, we start with Eqn. 9.7.5 of the text

$$\alpha(\hbar\omega) = \frac{\omega}{n_r c\epsilon_0} \frac{|M_{21}|^2 \gamma}{(E_2 - E_1 - \hbar\omega)^2 + \gamma^2} (N_1 - N_2) \quad (2.4)$$

Since we are only considering the case where the first energy level is filled, we have $N_1 = N_D$ and $N_2 = 0$. Also, by the definition of the line width, $\gamma = \Gamma/2$. A plot of the absorption is shown in Fig. 2.1(b).

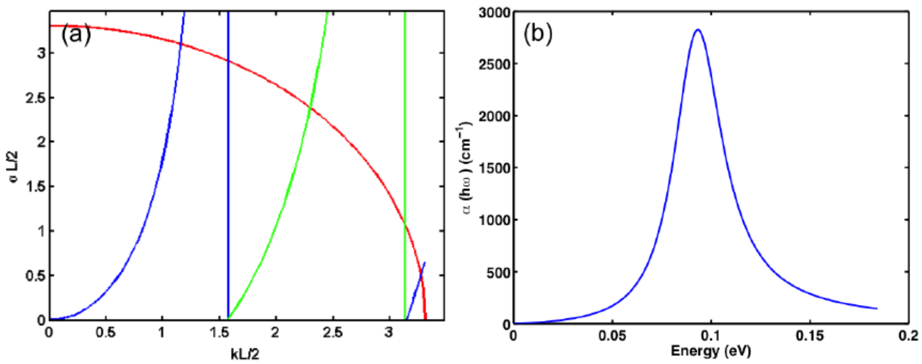

Figure 2.1: Finite well energy solutions and inter band absorption coefficients for AlGaAs system.

Problem 3)

The current density at threshold J_{th} in a semiconductor laser is related to the carrier density by the following relation

$$J_{th} = \left(\frac{q\,d}{\eta_i \tau_e}\right) N_{th}$$

The carrier density at threshold is given by

$$N_{th} = \left(\frac{\gamma}{g' L\Gamma}\right) + N_{tr}$$

where γ is the total loss per pass, and Γ is the beam confinement factor. We can use the approximate relations

$$V = \frac{2\pi}{\lambda_0} d\sqrt{n_1^2 - n_2^2} = 0.885$$

$$\Gamma \approx \frac{V^2}{2 + V^2} = 0.2814$$

from which

$$N_{th} = \frac{\gamma}{g' L\Gamma} + N_{tr} = 4.7\times10^{17} + 1.2\times10^{18} = 1.67\times10^{18}\,\text{cm}^{-3}$$

and

$$J_{th} = \left(\frac{q\,d}{\eta_i \tau_e}\right) N_{th} = 4.216 \times 10^{-16} \times 1.67 \times 10^{18} = 704 \text{ A/cm}^2$$

Problem 4)

The threshold carrier density for this case is

$$N_{th} = \frac{\gamma}{g' L \Gamma} + N_{tr} = \frac{143}{6 \times 10^{-16} \times 300 \times 10^{-4} \times 1.8 \times 10^{-2}} + 1.2 \times 10^{18}$$
$$= 4.4 \times 10^{18} + 1.2 \times 10^{18} = 5.6 \times 10^{18} \text{cm}^{-3}$$

Using the same model as above for the threshold current

$$J_{th} = \left(\frac{q\,d}{\eta_i \tau_e}\right) N_{th} = d\, \frac{1.602 \times 10^{-19}}{0.95 \times 4 \times 10^{-9}} \times 5.6 \times 10^{18}$$
$$= d \times 236 \times 10^{6} \text{ A/cm}^2$$

Now, we need to realize that here the active region *d* should be the width of the quantum well region. The simplest implementation of the structure could be to form a step-index optical waveguide (e.g., a core with 20% and a cladding with 60% Al mole fraction) with a GaAs quantum well in the middle. In this course we have seen before quantum well prototypes with well width 10 nm = 10^{-6} cm. With this choice we obtain a threshold current density

$$J_{th} = \left(\frac{q\,d}{\eta_i \tau_e}\right) N_{th} = 10^{-6} \times 236 \times 10^{6} = 236 \text{ A/cm}^2$$

which is about three times smaller than the threshold of Problem 3. This is consistent with the expectation that a quantum well laser should be more efficient than the bulk DH device. In order to be fully consistent with the confinement factor $\Gamma = 1.8 \times 10^{-2}$, the width of the core in the waveguide could be chosen so that the optical power is distributed accordingly in the well, the core and the cladding layers and one could analyze the shape of the optical power, assuming for instance a monomode optical profile, to estimate more precisely the optimal width of the well. We don't need to go that far for our purposes, but just consider that, for a wider quantum well, more carriers need to be injected, thus requiring higher current, in order to reach the necessary carrier density in the active volume. The opposite occurs for a narrower well. Then, for a fixed core layer of the waveguide, the confinement factor can be fine tuned by changing the width of the quantum well. In the expression for the carrier density at threshold there are two competing factors: the differential gain, which tends to be higher for a quantum well laser with respect to a bulk DH device, and the confinement factor, which tends to be lower.